The central differencing scheme failed to produce a reasonable result with the same grid resolution. The upwind scheme produces a much more realistic solution that is, however, not very close to the exact solution near boundary B.

Table 5.7

Node	*Distance*	*Finite volume solution*	*Analytical solution*	*Difference*	*Percentage error*
1	0.1	0.9998	1.0000	0.0002	0.02
2	0.3	0.9987	0.9999	0.001	0.13
3	0.5	0.9921	0.9999	0.008	0.79
4	0.7	0.9524	0.9994	0.047	4.71
5	0.9	0.7143	0.9179	0.204	22.18

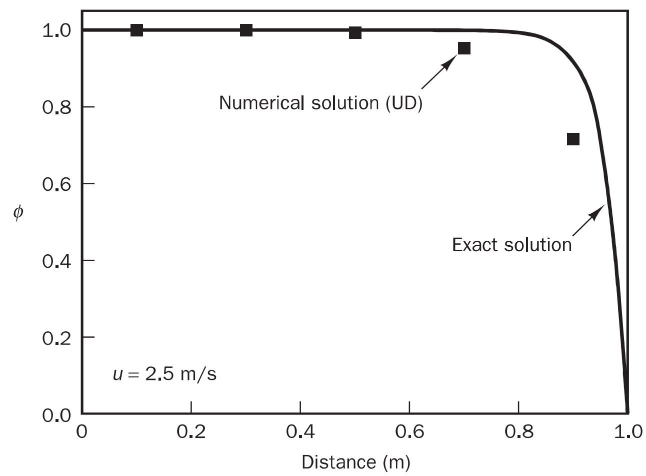


Figure 5.13 Comparison of the upwind difference numerical results and the analytical solution for Case 2

5.6.1 Assessment of the upwind differencing scheme

Conservativeness: The upwind differencing scheme utilises consistent expressions to calculate fluxes through cell faces: therefore it can be easily shown that the formulation is conservative.

Boundedness: The coefficients of the discretised equation are always positive and satisfy the requirements for boundedness. When the flow satisfies continuity the term $(F_e - F_w)$ in a_P (see (5.31)) is zero and gives $a_P = a_W + a_E$, which is desirable for stable iterative solutions. All the coefficients are positive and the coefficient matrix is diagonally dominant, hence no 'wiggles' occur in the solution.

Transportiveness: The scheme accounts for the direction of the flow so transportiveness is built into the formulation.

Accuracy: The scheme is based on the backward differencing formula so the accuracy is only first-order on the basis of the Taylor series truncation error (see Appendix A).

Because of its simplicity the upwind differencing scheme has been widely applied in early CFD calculations. It can be easily extended to multi-dimensional problems by repeated application of the upwind strategy embodied in the coefficients of (5.31) in each co-ordinate direction. A major drawback of the scheme is that it produces erroneous results when the flow is not aligned with the grid lines. The upwind differencing scheme causes the distributions of the transported properties to become smeared in such problems. The resulting error has a diffusion-like appearance and is referred to as **false diffusion**. The effect can be illustrated by calculating the transport of scalar property ϕ using upwind differencing in a domain where the flow is at an angle to a Cartesian grid.

In Figure 5.14 we have a domain where $u = v = 2$ m/s everywhere so the velocity field is uniform and parallel to the diagonal (solid line) across the grid. The boundary conditions for the scalar are $\phi = 0$ along the south and east boundaries, and $\phi = 100$ on the west and north boundaries. At the first and the last nodes where the diagonal intersects the boundary a value of 50 is assigned to property ϕ.

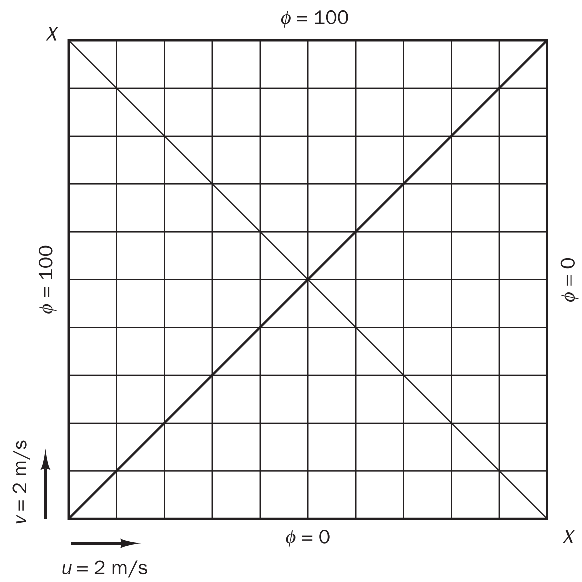
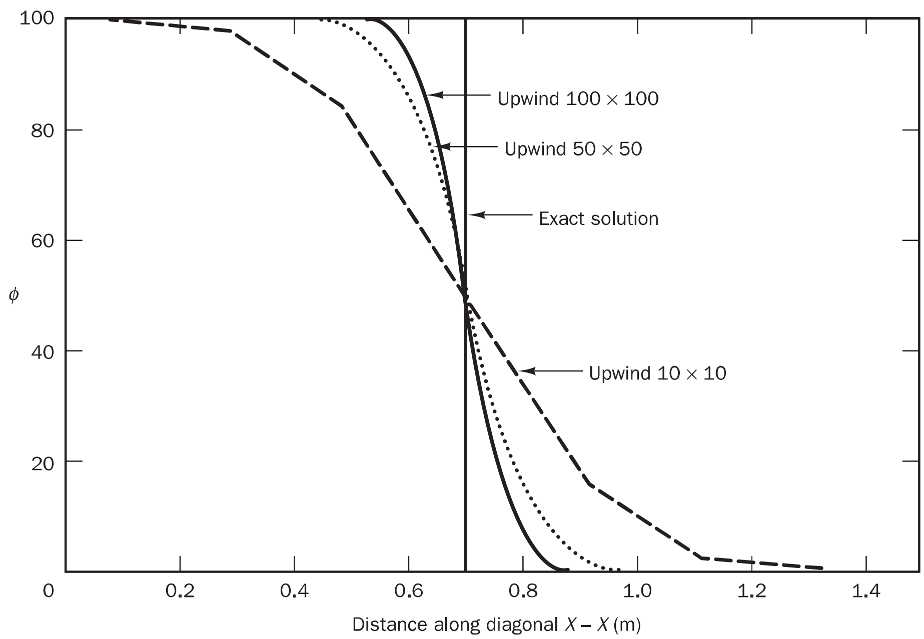


Figure 5.14 Flow domain for the illustration of false diffusion

To identify the false diffusion due to the upwind scheme, a pure convection process is considered without physical diffusion. There are no source terms for ϕ and a steady state solution is sought. The correct solution is known in this case. As the flow is parallel to the solid diagonal the value of ϕ at all nodes above the diagonal should be 100 and below the diagonal it should be zero. The degree of false diffusion can be illustrated by calculating the distribution of ϕ and plotting the results along the diagonal (X–X). Since there is no physical diffusion the exact solution exhibits a step change of ϕ from 100 to zero when the diagonal X–X crosses the solid diagonal. The calculated results for different grids are shown in Figure 5.15 together with the exact solution. The numerical results show badly smeared profiles.

The error is largest for the coarsest grid, and the figure shows that refinement of the grid can, in principle, overcome the problem of false diffusion. The results for 50×50 and 100×100 grids show profiles that are closer to the exact solution. In practical flow calculations, however, the degree of grid refinement required to eliminate false diffusion can be prohibitively expensive. Trials have shown that, in high Reynolds number

Figure 5.15



flows, false diffusion can be large enough to give physically incorrect results (Leschziner, 1980; Huang *et al.*, 1985). Therefore, the upwind differencing scheme is not entirely suitable for accurate flow calculations and considerable research has been directed towards finding improved discretisation schemes.

5.7 The hybrid differencing scheme

The hybrid differencing scheme of Spalding (1972) is based on a combination of central and upwind differencing schemes. The central differencing scheme, which is second-order accurate, is employed for small Peclet numbers ($Pe < 2$) and the upwind scheme, which is first-order accurate but accounts for transportiveness, is employed for large Peclet numbers ($Pe \geq 2$). As before, we develop the discretisation of the one-dimensional convection–diffusion equation without source terms. This equation can be interpreted as a flux balance equation. The hybrid differencing scheme uses piecewise formulae based on the local Peclet number to evaluate the net flux through each control volume face. The Peclet number is evaluated at the face of the control volume. For example, for a west face,

$$Pe_w = \frac{F_w}{D_w} = \frac{(\rho u)_w}{\Gamma_w / \delta x_{WP}} \tag{5.35}$$

The hybrid differencing formula for the net flux per unit area through the west face is as follows:

$$\begin{aligned} q_w &= F_w \left[\frac{1}{2}\left(1 + \frac{2}{Pe_w}\right)\phi_W + \frac{1}{2}\left(1 - \frac{2}{Pe_w}\right)\phi_P \right] && \text{for } -2 < Pe_w < 2 \\ q_w &= F_w \phi_W && \text{for } Pe_w \geq 2 \qquad (5.36) \\ q_w &= F_w \phi_P && \text{for } Pe_w \leq -2 \end{aligned}$$